講義資料

http://conf.msl.titech.ac.jp/Lecture/

統計力学・半導体

課題

バースタイン･モスシフト (縮退半導体の E_F) ΔE_gをキャリア濃度 Ne の関数としてグラフに描け。有効質量は自由電子の質量とし、横軸 Neは対数プロットせよ

$$\Delta E_g^{BM} = \frac{h^2}{m_{de}}\left(\frac{3N_e}{16\sqrt{2}\pi}\right)^{2/3}$$

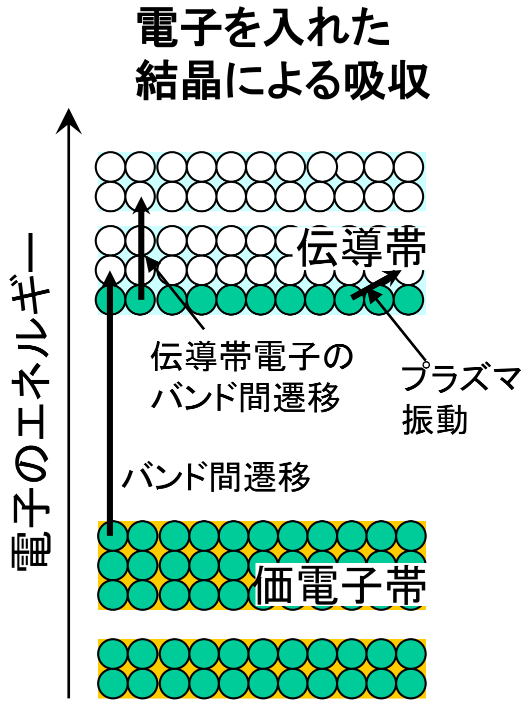
PowerPoint 等 のプレゼンテーションファイルにして提出

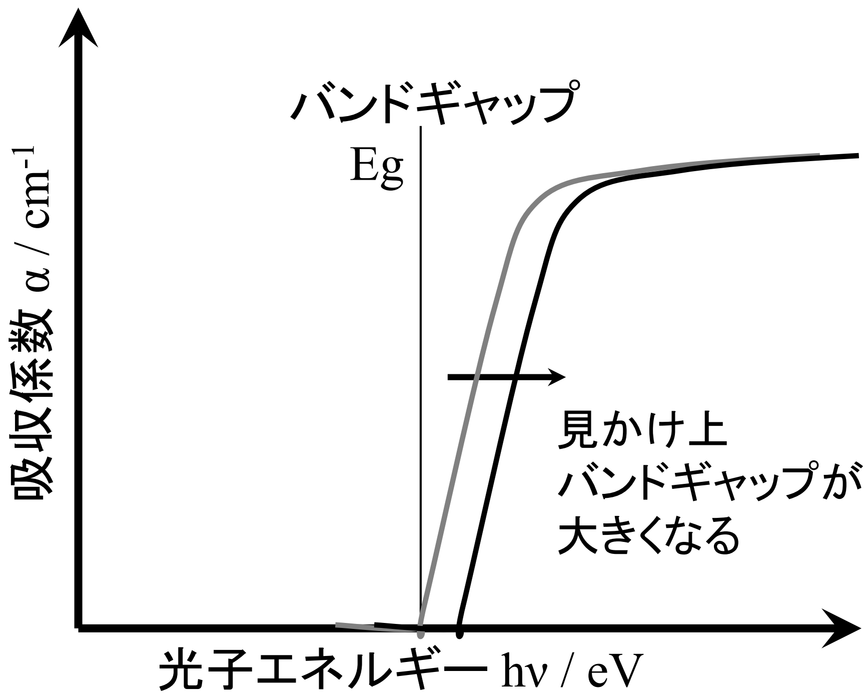
期限: 今日の17:00までに

できたところまでで可

高ドープ半導体の光吸収

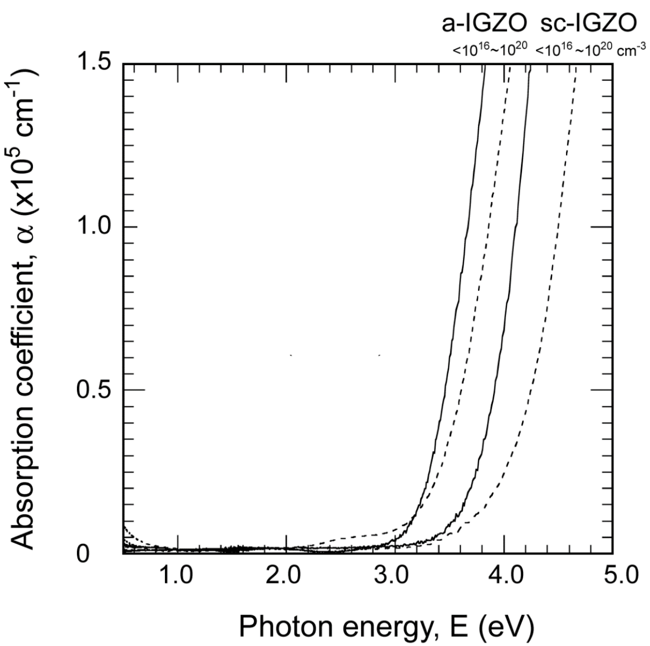
バンドフィリング効果 (Burstein-Moss shift)

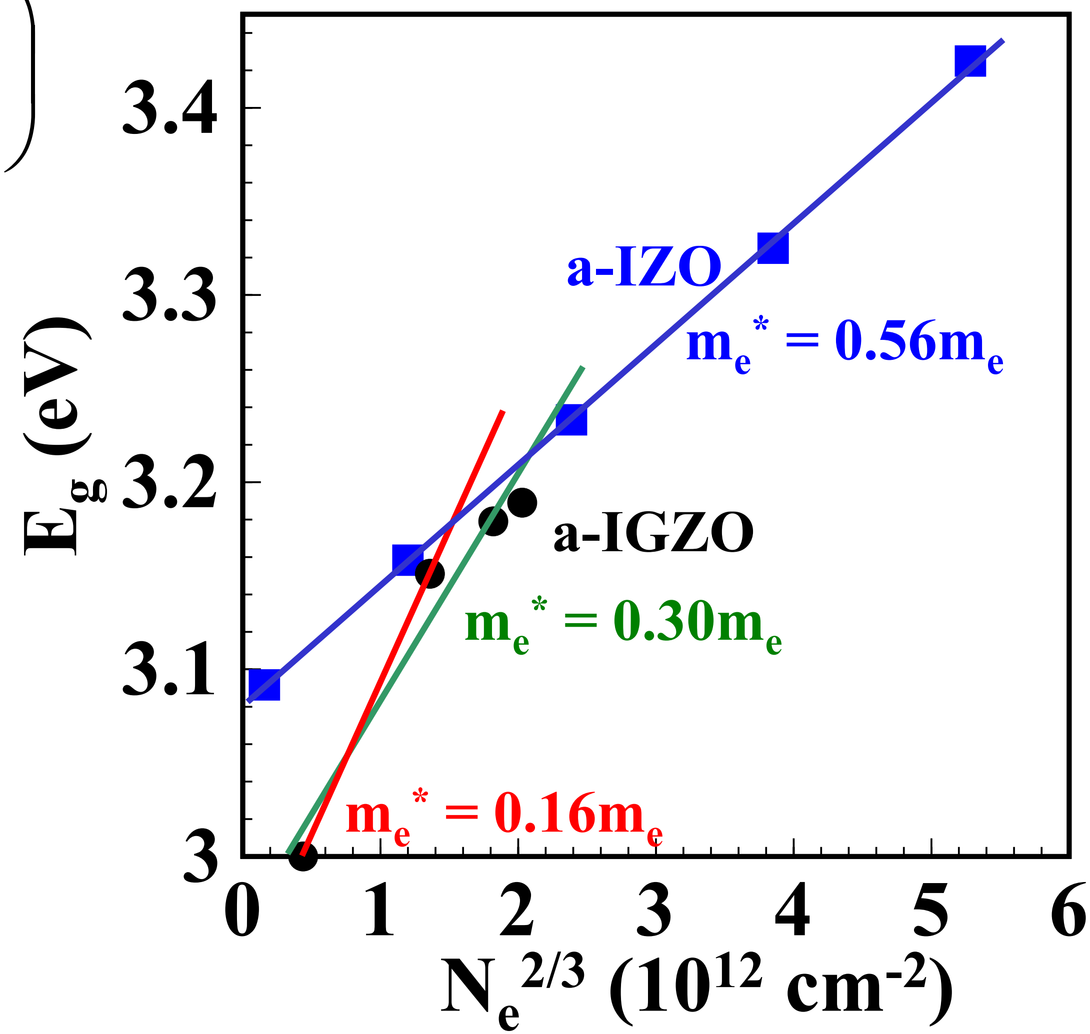




バンドフィリング効果(BMシフト)

$$\Delta E_g^{BM} = \frac{h^2}{m_{de}}\left(\frac{3N_e}{16\sqrt{2}\pi}\right)^{2/3}$$





半導体: キャリア輸送

移動度とは？

$$\sigma = \mathrm{e}n\mu$$

Definition in solid-state physics

一電子の運動方程式　$F = m_e\left(\frac{d}{dt}v - \frac{1}{\tau}v\right) = qE$

m_e: 有効質量

τ　: 運動量緩和時間 (散乱時間)

電子が持っている運動量が散乱を受けて
0 になる平均時間

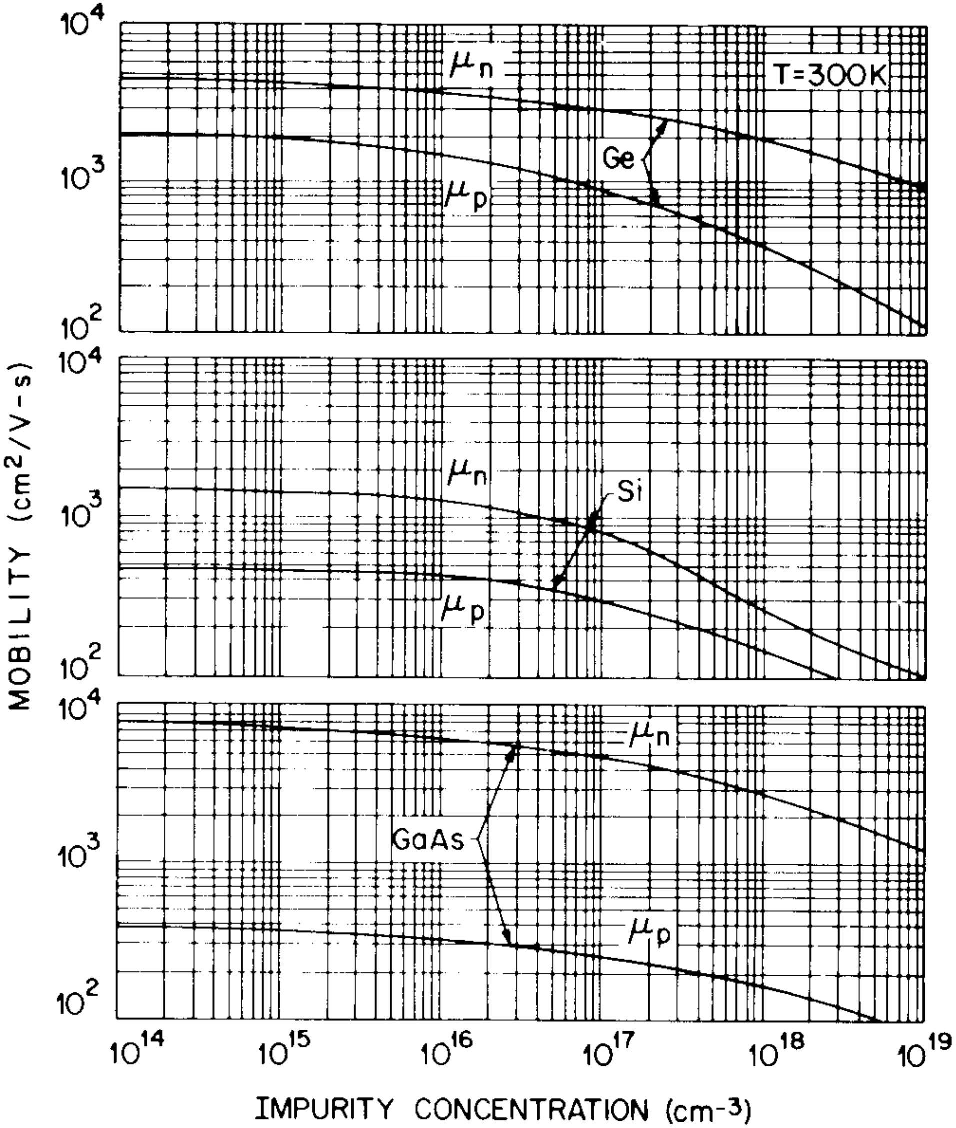
定常状態での速度　$v = \mu E = \frac{e}{m_e}\tau E$

ドリフト速度　v_d : 電界によって駆動される速度

⇔ 熱速度、Fermi速度、拡散速度

ドリフト移動度　$\mu_d = \frac{v_d}{E} = \frac{e}{m_e}\tau$

Mobility vs. doping conc.



S.M. Sze, Physics of Semiconductor Devices

移動度

半導体評価技術

河東田　隆　編著

$$\frac{1}{\tau(x)} = \sum \frac{1}{\tau_i(x)}$$

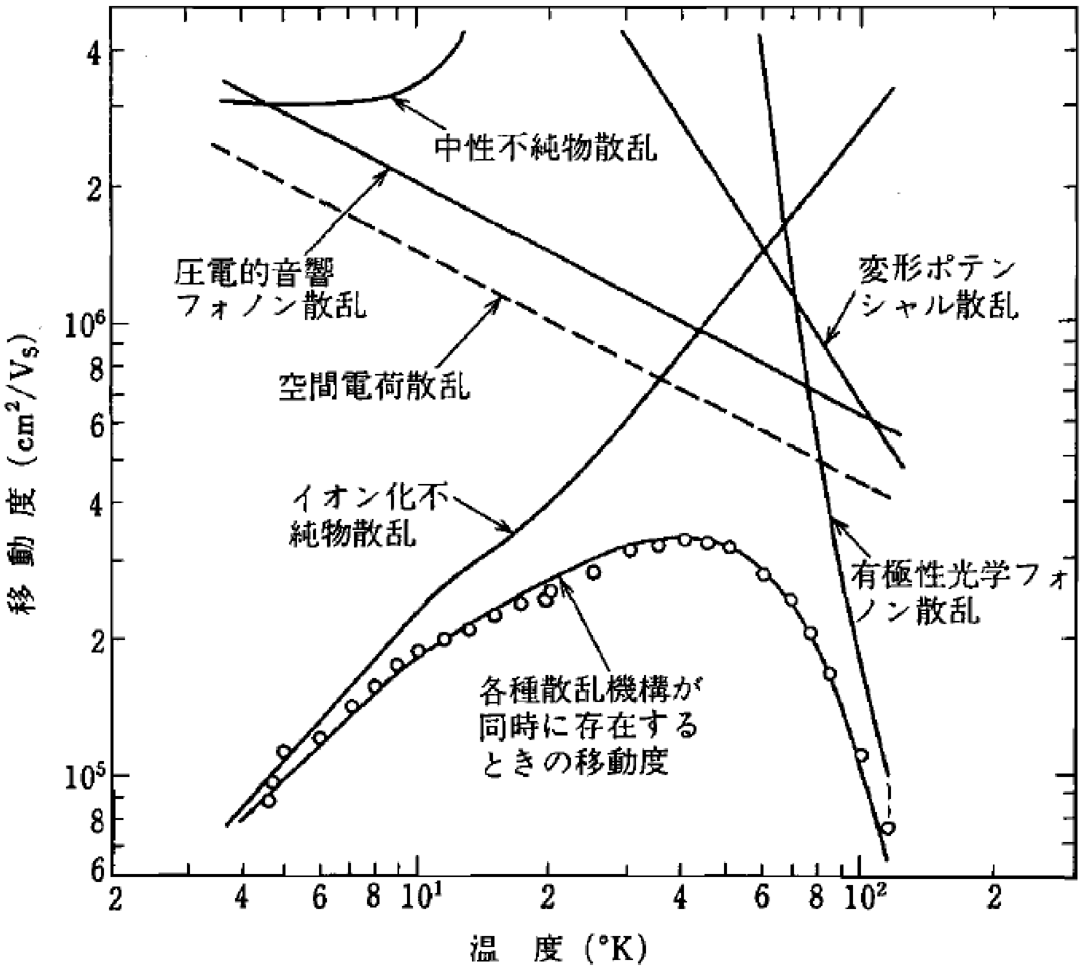
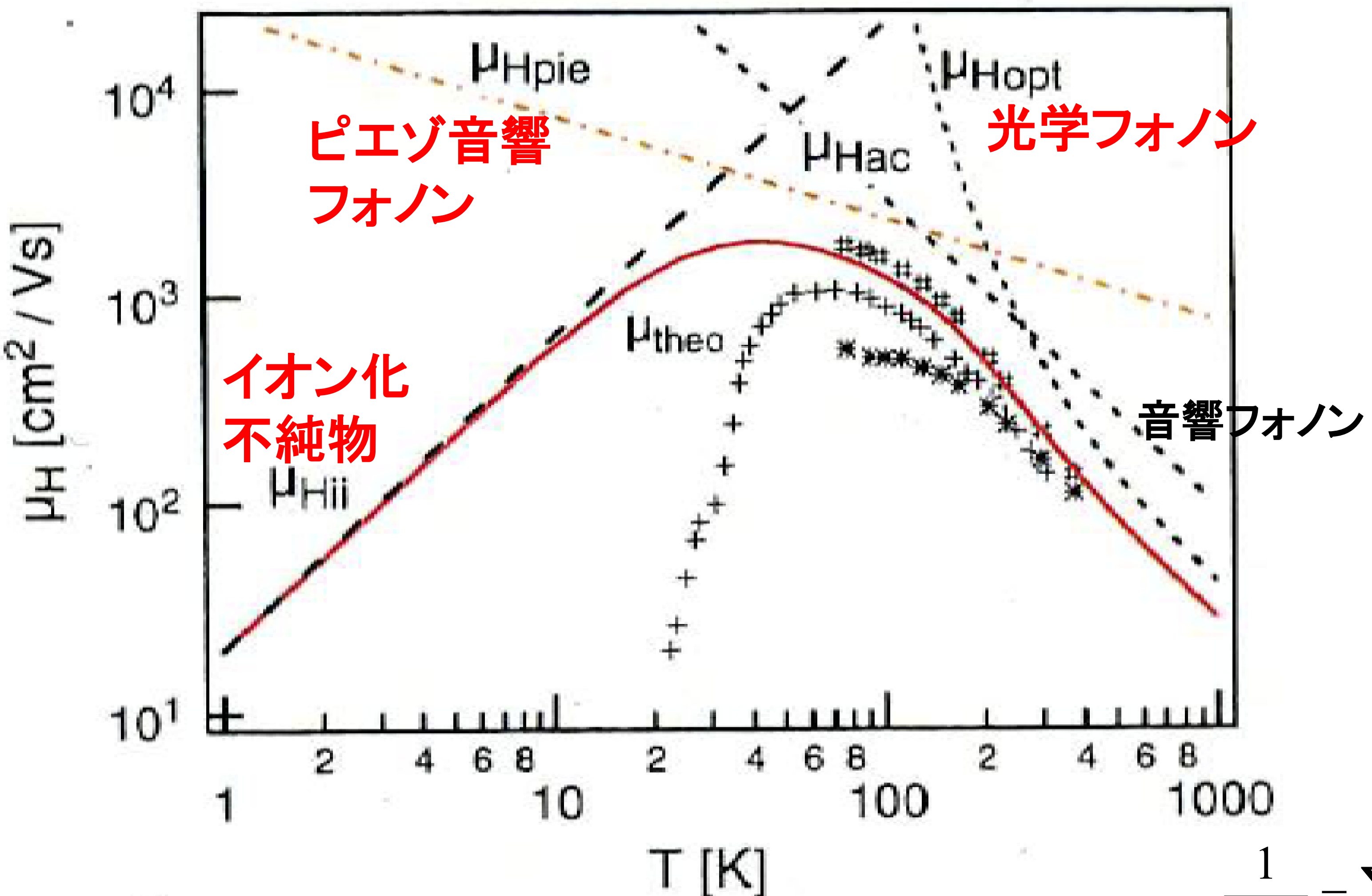


図 6.5　各種散乱機構による電子移動度．GaAs の場合，実線および破線は計算値，○は実測値，各種散乱機構が同時に存在するときの移動度の計算には，空間電荷散乱は考慮されていない．

ZnOの移動度



$$\frac{1}{\tau(x)} = \sum \frac{1}{\tau_i(x)}$$

K. Ellmer, Handbook of Transparent Conductors, Fig. 7.13, p.216, Ed. D.S. Ginley (Springer, New York, 2010)

P. Wagner and R. Helbig, J. Phys. Chem. Solids, 35 (1974) 327

散乱機構と移動度の温度依存性

$$\tau = \tau_0 \varepsilon^{r-1/2} \qquad \mu = \frac{e}{m_e}\langle\tau\rangle = \mu_0 T^s$$

音響フォノン散乱 (非縮退) $\tau = \tau_0 \varepsilon^{-1/2},\ \mu \propto T^{-3/2}$

音響フォノン散乱 (縮退) $\tau = \tau_0 \varepsilon^{-1/2},\ \mu \propto T^{-1}$

光学フォノン散乱 T<<θ_D,高ドープ $\tau = \tau_0 \varepsilon^{0},\ \langle\tau\rangle \propto \left[\exp(\hbar\omega_0/kT) - 1\right]$

光学フォノン散乱 T<<θ_D,低ドープ $\tau = \tau_0 \varepsilon^{0},\ \langle\tau\rangle \propto T^{1/2}$

イオン化不純物 (非縮退) $\tau = \tau_0 \varepsilon^{3/2},\ \mu \propto T^{3/2}$

イオン化不純物 (縮退) $\tau = \tau_0 \varepsilon^{3/2},\ \mu \propto T^{0}$

中性不純物 $\tau = \tau_0 \varepsilon^{0},\ \mu \propto T^{0}$

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TABLE 3.2. $\tau = \tau_0 (\varepsilon^*)^{r-1/2}$

Scattering centers, r	τ_0	Notation used
Acoustical vibrations (phonon theory), r = 0	$\frac{9\pi}{4\sqrt{2}} \frac{\hbar^4 \omega^2 M}{C^2 a^3 (m^* kT)^{3/2}}$	ω – velocity of sound; M – atomic mass; C – Bloch constant; a – lattice parameter
Acoustical vibrations (deformation potential theory), r = 0	$\frac{\pi \hbar^4 C_{ll}}{\sqrt{2} E_1^2 (m^* kT)^{3/2}}$	C_{ll} – elastic constant for longitudinal vibrations; $E_1 = \Omega_0 dE_0/d\Omega$; E_0 – energy of allowed band edge; Ω_0 – initial volume of unit cell before deformation
Optical vibrations ($T \ll \theta_D$) in heavily doped crystals, r = 1/2	$\frac{a^3 M}{2\pi\sqrt{2m^*}} \frac{(\hbar\omega_0)^{3/2}}{(\gamma Z e^2)^2} \times \left[\exp\left(\frac{\hbar\omega_0}{kT}\right) - 1\right](1-f_0)$	ω_0 – limiting frequency of longitudinal optical vibrations; Ze – ion charge; γ – factor representing the polarizability of ions; f – Fermi function; θ_D – Debye temperature
Optical vibrations ($T \ll \theta_D$) in lightly	$\frac{a^3 M}{2\pi\sqrt{2m^*}} \frac{(\hbar\omega_0)^{3/2}}{(\gamma Z e^2)^2} \times$	

TABLE III. Approximate ϵ and T dependencies for electron-scattering mechanisms.

Scattering mechanism	Energy dependence of τ	τ	Temperature dependence of μ^{nondegen}	μ^{degen}
Intravalley acoustic phonons	$\epsilon^{-1/2}$	T^{-1}	$T^{-3/2}$	T^{-1}
Intervalley optical phonons	$\epsilon^{-1/2}$	T^{-1}	$T^{-3/2}$	T^{-1}
Ionized impurities	$\epsilon^{3/2}$	T^{0}	$T^{3/2}$	T^{0}
Alloy disorder	$\epsilon^{-1/2}$	T^{0}	$T^{-1/2}$	T^{0}
Neutral impurities	ϵ^{0}	T^{0}	T^{0}	T^{0}

Heavily doped semiconductor, P.86

A model for the high-temperature transport properties of heavily doped n-type silicon-germanium alloys, JAP 69 (1991) 331 Fig. 3

Hall効果と磁気抵抗効果

Hall効果

電荷 q (電子: -e, 正孔: +e)

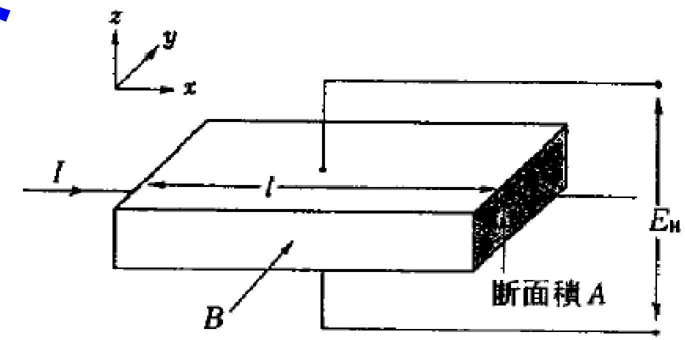


図 3·24 Hall 効果の実験

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) \mathbf{v} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_x = q(E + B v_y)$$

$$m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_z = 0 \qquad m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_y = q(-B v_x)$$

$$v_x = -\frac{e\tau}{m^*} E - \omega_c \tau v_y$$

$$v_y = \omega_c \tau v_x \qquad v_z = 0 \qquad \omega_c = eB/m^* c$$

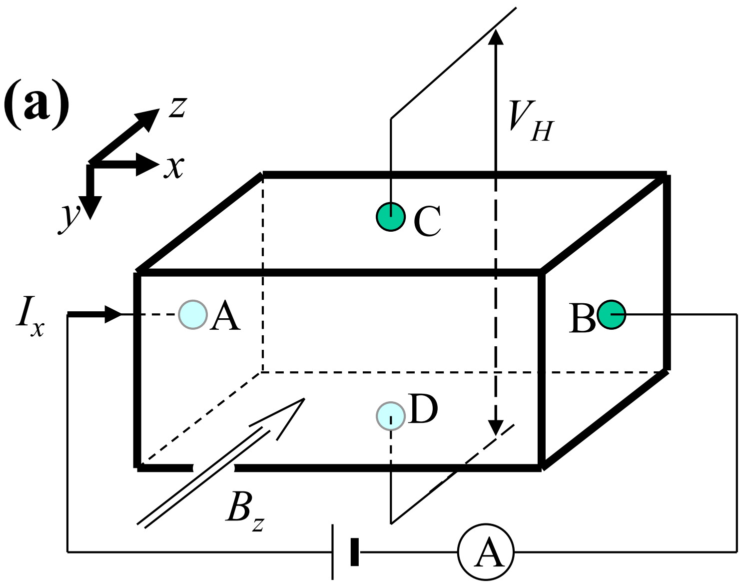
$$m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_y = q(E_{Hall} - B v_x)$$

$$E_{Hall} = \frac{H}{c} v_x = \frac{qH\tau}{m^* c} E \qquad j_x = \frac{nq^2\tau}{m^*} E_x$$

$$\boxed{R_H = \frac{E_{Hall}}{j_x B} = \frac{1}{nq}}$$

キャリア極性(R_Hの符号)、キャリア濃度n_{Hall}、移動度μ_{Hall}

六端子Hallバー



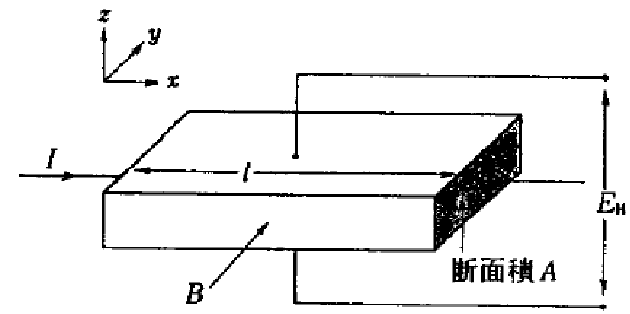
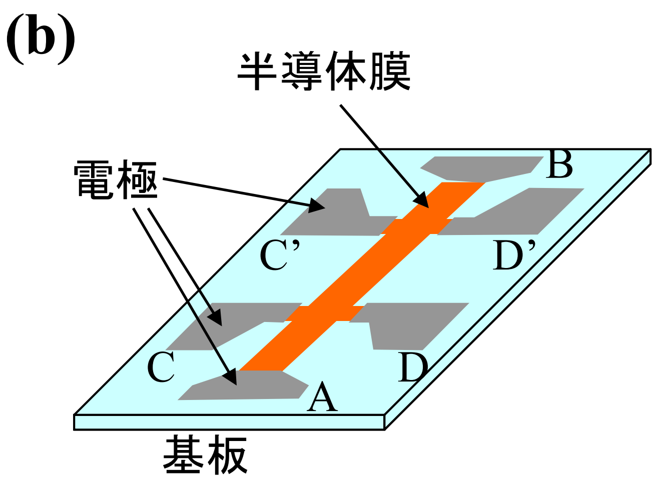


図 3·24 Hall 効果の実験

四端子測定で抵抗率を測定
電圧端子(C,D)位置を確定する必要



- パターニングが必要
- キャリアの伝導経路をかなり限定できる
- 四端子測定で抵抗率を測定
- 複数のHall電圧端子の組み合わせで信頼性を上げる

磁場反転、電流反転測定で誤差を相殺

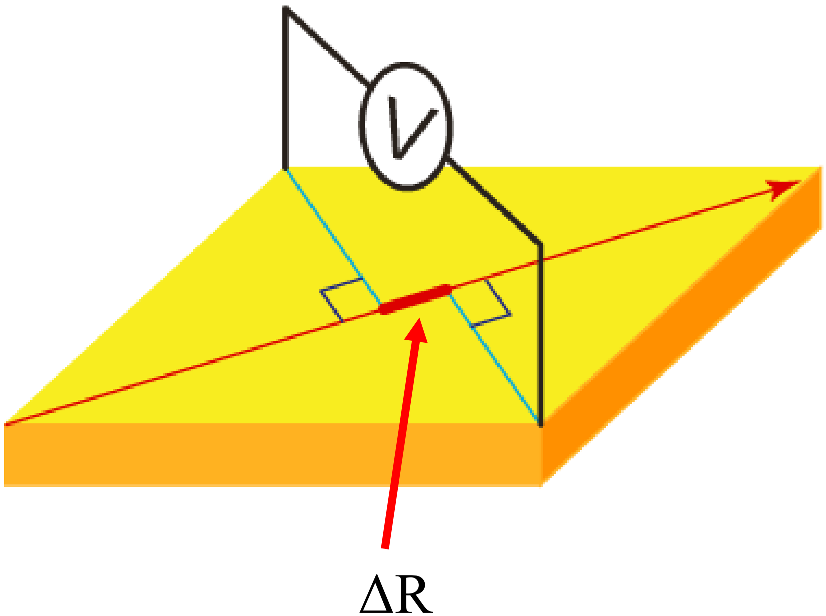
Hall電圧電極がずれている
　試料のΔRI だけ電圧がずれる

$$V_{obs}{}^{+} = BIR_{Hall}/t + I\Delta R$$

・磁場を反転させて測定

$$V_{obs}{}^{-} = -BIR_{Hall}/t + I\Delta R$$

=> $(V_{obs}{}^{+} - V_{obs}{}^{-})/2 = -BIR_{Hall}/t$



Hall effect

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

$-e$: Electron charge, Under E_x and B_z

Motion of dynamics $$m_e^*\left(\frac{d\boldsymbol{v_i}}{dt}+\frac{\boldsymbol{v_i}}{\tau}\right)=-e(\boldsymbol{E}+\boldsymbol{v_i}\times\boldsymbol{B})$$

Average velocity $\langle\boldsymbol{v}\rangle=\sum\boldsymbol{v_i}/n$

$$m_e^*\left(\frac{d\langle\boldsymbol{v}\rangle}{dt}+\frac{\langle\boldsymbol{v}\rangle}{\tau}\right)=-e(\boldsymbol{E}+\langle\boldsymbol{v}\rangle\times\boldsymbol{B})$$

$$m_e^*\langle\boldsymbol{v}\rangle_x=-e\tau\left(E_x+\langle\boldsymbol{v}\rangle_y B_z\right)$$

$$m_e^*\langle\boldsymbol{v}\rangle_y=-e\tau\left(E_y-\langle\boldsymbol{v}\rangle_x B_z\right)$$

$$m_e^*\langle\boldsymbol{v}\rangle_z=-e\tau E_z$$

$$\langle\boldsymbol{v}\rangle_x=-\frac{e\tau}{m_e^*}\frac{E_x+\frac{e\tau}{m_e^*}B_zE_y}{1+\left(\frac{e\tau}{m_e^*}\right)^2{B_z}^2}$$

$$\langle\boldsymbol{v}\rangle_y=-\frac{e\tau}{m_e^*}\frac{E_y-\frac{e\tau}{m_e^*}B_zE_x}{1+\left(\frac{e\tau}{m_e^*}\right)^2{B_z}^2}$$

$$\langle\boldsymbol{v}\rangle_x=-\frac{e\tau}{m_e^*}E_z$$

Hall effect

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

Current $$\boldsymbol{J} = -en\langle \boldsymbol{v} \rangle = \frac{e^2 n\tau}{m_e^*}\begin{pmatrix} \dfrac{1}{1+\left(\dfrac{e\tau}{m_e^*}\right)^2 {B_z}^2} & \dfrac{\left(\dfrac{e\tau}{m_e^*}\right)B_z}{1+\left(\dfrac{e\tau}{m_e^*}\right)^2 {B_z}^2} & 0 \\ -\dfrac{\left(\dfrac{e\tau}{m_e^*}\right)B_z}{1+\left(\dfrac{e\tau}{m_e^*}\right)^2 {B_z}^2} & \dfrac{1}{1+\left(\dfrac{e\tau}{m_e^*}\right)^2 {B_z}^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$= \sigma_{xx}\left[\boldsymbol{E} + \left(\frac{e\tau}{m_e^*}\right)^2 \boldsymbol{B}(\boldsymbol{E}\cdot\boldsymbol{B}) + \frac{e\tau}{m_e^*}(\boldsymbol{E}\times\boldsymbol{B})\right]$$

$\omega_c = \left(\dfrac{e}{m_e^*}\right)B_z$: cyclotron frequency

Hall効果

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

Hall効果測定の条件 $J_y = 0$ $(\mu = \frac{e\tau}{m_e^*})$ **より**

$$en\mu \begin{pmatrix} \frac{1}{1+\mu^2 {B_z}^2} & \frac{\mu B_z}{1+\mu^2 {B_z}^2} \\ -\frac{\mu B_z}{1+\mu^2 {B_z}^2} & \frac{1}{1+\mu^2 {B_z}^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{\sigma_0}{1+\mu^2 {B_z}^2} \begin{pmatrix} 1 & \mu B_z \\ -\mu B_z & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

$$E_x = \frac{1}{\sigma_0} \frac{J_x \sigma_{xx}}{\sigma_{xx}\sigma_{yy} - \sigma_{yx}\sigma_{yx}} = \frac{1}{\sigma_0} J_x$$

$$E_y = -\frac{1}{\sigma_0} \frac{J_x \sigma_{yz}}{\sigma_{xx}\sigma_{yy} - \sigma_{yx}\sigma_{yx}} = \frac{1}{\sigma_0} \mu B_z J_x = \frac{1}{en} B_z J_x = R_H B_z J_x$$

$$\boldsymbol{R_H = -\frac{E_y}{B_z J_x}} = -\frac{V_H}{W} \frac{Wd}{I_x} \frac{1}{B_z} = -\boldsymbol{\frac{V_H}{I_x} \frac{d}{B_z}} = -\frac{1}{en} \text{ (for electron)}$$

Hall effect: Two-carrier model

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

Current $\quad \boldsymbol{J} = -e\langle \boldsymbol{v} \rangle = \frac{e^2 n\tau}{m_e^*} \begin{pmatrix} \frac{1}{1+\left(\frac{e\tau}{m_e^*}\right)^2 {B_z}^2} & \frac{\left(\frac{e\tau}{m_e^*}\right) B_z}{1+\left(\frac{e\tau}{m_e^*}\right)^2 {B_z}^2} & 0 \\ -\frac{\left(\frac{e\tau}{m_e^*}\right) B_z}{1+\left(\frac{e\tau}{m_e^*}\right)^2 {B_z}^2} & \frac{1}{1+\left(\frac{e\tau}{m_e^*}\right)^2 {B_z}^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$

Two carrier model

$$\boldsymbol{J} = \boldsymbol{J_e} + \boldsymbol{J_h} = (\sigma_e + \sigma_h)\boldsymbol{E} + (-\sigma_e \mu_e + \sigma_h \mu_h)\boldsymbol{E} \times \boldsymbol{B}$$

$$\rho_{xx}(B) = \frac{1}{e} \frac{(n_h \mu_h + n_e \mu_e) + (n_h \mu_h + n_e \mu_e)\mu_h \mu_e B^2}{(n_h \mu_h + n_e \mu_e)^2 + (n_h - n_e)^2 {\mu_h}^2 {\mu_e}^2 B^2}$$

$$\rho_{yx}(B) = B \frac{1}{e} \frac{\left(n_h {\mu_h}^2 - n_e {\mu_e}^2\right) + (n_h - n_e){\mu_h}^2 {\mu_e}^2 B^2}{(n_h \mu_h + n_e \mu_e)^2 + (n_h - n_e)^2 {\mu_h}^2 {\mu_e}^2 B^2}$$

Hall effect: Two-carrier model with $n_e \sim n_h$

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

Two carrier model

$$\rho_{xx}(B) = \frac{1}{e}\frac{(n_h\mu_h+n_e\mu_e)+(n_h\mu_h+n_e\mu_e)\mu_h\mu_e B^2}{(n_h\mu_h+n_e\mu_e)^2+(n_h-n_e)^2\mu_h{}^2\mu_e{}^2B^2}$$

$$\rho_{yx}(B) = B\,\frac{1}{e}\frac{\left(n_h\mu_h{}^2-n_e\mu_e{}^2\right)+(n_h-n_e)\mu_h{}^2\mu_e{}^2B^2}{(n_h\mu_h+n_e\mu_e)^2+(n_h-n_e)^2\mu_h{}^2\mu_e{}^2B^2}$$

$\boldsymbol{n_e \sim n_h \sim n}$

$$\rho_{xx}(B) = \frac{1}{en}\frac{1+\mu_h\mu_e B^2}{\mu_h+\mu_e}$$

$$\rho_{yx}(B) = B\,\frac{1}{en}\frac{\mu_h-\mu_e}{\mu_h+\mu_e}$$

Hall effect: Two-carrier model with $\mu_h \sim \mu_e$

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

Two carrier model

$$\rho_{xx}(B) = \frac{1}{e}\frac{(n_h\mu_h+n_e\mu_e)+(n_h\mu_h+n_e\mu_e)\mu_h\mu_e B^2}{(n_h\mu_h+n_e\mu_e)^2+(n_h-n_e)^2\mu_h{}^2\mu_e{}^2B^2}$$

$$\rho_{yx}(B) = B\,\frac{1}{e}\frac{\left(n_h\mu_h{}^2-n_e\mu_e{}^2\right)+(n_h-n_e)\mu_h{}^2\mu_e{}^2B^2}{(n_h\mu_h+n_e\mu_e)^2+(n_h-n_e)^2\mu_h{}^2\mu_e{}^2B^2}$$

$\mu_h \sim \mu_e \sim \mu$

$$\boldsymbol{\rho_{xx}(B)} = \frac{1}{e\mu(n_h+n_e)}\frac{1+\mu^2B^2}{1+(n_h-n_e)^2/(n_h+n_e)^2\mu^2B^2}$$

$$\sim\rho_0(1+\mu^2B^2-(n_h-n_e)^2/(n_h+n_e)^2\mu^2B^2)$$

$$\boldsymbol{\sim\rho_0\left(1+\mu^2B^2\right)}$$

$$\rho_{yx}(B) = B\,\frac{1}{e(n_h+n_e)}\frac{1+\mu^2B^2}{1+(n_h-n_e)^2/(n_h+n_e)^2\mu^2B^2} \sim B\,\frac{1}{e(n_h+n_e)}$$

Hall effect: Two-carrier model

太田英二、坂田亮著、半導体の電子物性光学、培風館 (2005)

When electrons and holes coexist

$$\boldsymbol{J_e} = \sigma_e \boldsymbol{E} - \sigma_e \mu_e \boldsymbol{E} \times \boldsymbol{B}$$

$$\boldsymbol{J_h} = \sigma_h \boldsymbol{E} + \sigma_h \mu_h \boldsymbol{E} \times \boldsymbol{B}$$

$$\boldsymbol{J} = \boldsymbol{J_e} + \boldsymbol{J_e} = (\sigma_e + \sigma_h)E + (-\sigma_e \mu_e + \sigma_h \mu_h)\boldsymbol{E} \times \boldsymbol{B}$$

$$R_H = \frac{n_h {\mu_h}^2 - n_e {\mu_e}^2}{e(n_h \mu_h + n_e \mu_e)^2}$$

(i) Only holes: $\boldsymbol{n_e = 0}$ => $\boldsymbol{R_H = \frac{1}{e n_h}}$

(ii) Same mobility: $\boldsymbol{\mu_h = \mu_e = \mu}$ => $\boldsymbol{R_H = \frac{n_h - n_e}{e(n_h + n_e)^2}}$

(iii) Nearly intrinsic: $\boldsymbol{n_h \sim n_e \sim n_i}$ => $\boldsymbol{R_H = \frac{1 - \mu_e/\mu_h}{e n_i (1 + \mu_e/\mu_h)}}$